The CYRUS Global Business Perspectives (CGBP)

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The CYRUS Global Business Perspectives (CGBP)

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Announcements

- CIK 2022 Conference – Location and dates will be announced later.
- CIK 2021 Conference – October 20th - 22nd, 2021, Online, in collaboration with SINGEP, Br
- CIK 2020 Conference – October 1st - 3rd 2020, Online, in collaboration with SINGEP, Brazil
- CIK 2019 Conference – April 17th - 21st 2019, MIT, Cambridge, USA
- CIK 2018 Conference – March 4th - 7th 2018, ESCA and UM5, Casablanca, Morocco
- CIK 2017 Conference – April 14th - 16th 2017, MIT, Cambridge, USA
- CIK 2016 Conference – March 15th - 17th 2016, The American University in Cairo, Egypt
- CIK 2015 Conference – April 24 - 26th 2015, Harvard University, Cambridge, USA
- CIK 2014 Conference – January 9th - 11th 2014, Hult International Business, Dubai, UAE
- CIK 2012 Conference – October 15th - 17th 2012, Hult International Business, Cambridge MA
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- Guidelines for submission to ***The CYRUS Global Business Perspectives*** CGBP.

The flagship journal of the CYRUS Institute of Knowledge

The CYRUS Global Business Perspectives (CGBP)

Purpose

The CYRUS Global Business Perspectives (CGBP) is a refereed interdisciplinary journal. The editorial objective is to create opportunities for scholars and practitioners to share theoretical and applied knowledge. The subject fields are management sciences, economic development, sustainable growth, and related disciplines applicable globally. CGBP provides a platform for the dissemination of high-quality research. We welcome contributions from researchers in academia and practitioners in broadly defined areas of management sciences, economic development, and sustainable growth.

Scope

The Journal's scope includes, but is not limited to, the following:

- Business Development and Governance
- Entrepreneurship
- Ethics and Social Responsibility
- International Business and Cultural Issues
- International Economics
- International Finance
- Innovation and Development
- Institutions and Development
- Leadership and Cultural Characteristics

- Natural Resources and Sustainable Development
- Organization and Cultural Issues
- Strategy and Development
- Women and Business Development

Reasons for choosing the CYRUS Global Business Perspectives

During this historical time, CGBP intends to offer a global perspective that is most urgently needed. There is no question that organizations and businesses that are capable of analyzing and applying advanced knowledge in management sciences and development are in high demand, and especially during transitional periods. It is an unusual time in different regions of the world, which requires active intellectual participation and contributions. It is the era of revolution in terms of communication, technology, and minds for billions of people. It is a time for intellectuals, entrepreneurs, and philanthropists to help enlighten minds and enrich the quality of life globally. CIK's vision, "to cultivate the discourse on human capital potentials for better living," is the appropriate response to current challenges and the CGBP is a platform for sharing the perspectives of scholars and practitioners with a global audience.

Our Strengths

CGBP members have a wealth of knowledge and global perspectives. First, most of the members possess a wealth of intellectual and experiential knowledge which is enhanced by their active involvement in business, consulting and scholarly research, and collegiate teaching. Second, most members possess an ethnic identity, language skills, and therefore, good global perspectives. Third, most of the CIK board of directors' members are well-known scholars, members of editorial boards of journals, and even editors. CGBP possesses depth, breadth, and a competitive edge to successfully manage the journal.

CGBP is committed to developing knowledge that positively contributes to the life of world citizens. *CIK is a charitable, educational, and scientific organization that has been in operation since 2011. It is a secular and nonpartisan organization.*

Submission Process

Authors will be assisted by an editorial board consisting of distinguished members from world-class institutions of higher learning, practice, and industry. We invite authors to submit their papers and case studies to Editor@Cyrusik.org. We will have a quick turn-around review process of fewer than two months. For submission guidelines, policies, and procedures please check CGBP. We intend to have special issues on themes that are within the scope of the Journal. Also, we will have invited (special) issues.

For more information please check the CGBP or CIK or contact us: Editor@Cyrusik.org; or Contact@Cyrusik.org. CYRUS Institute of Knowledge (CIK), Box 380003, Cambridge, MA 02238-0003, USA.

Editor's Introduction

Since its inception in 2012, the ***Cyrus Institute of Knowledge*** has held nine annual meetings. Six years ago, we published the first volume of its flagship journal titled "Cyrus Chronicle Journal (CCJ): Contemporary Economic and Management Studies in Asia and Africa," in conjunction with the 2016 annual conference. In 2021 after careful review, evaluation, and deliberation we concluded that a broader global perspective for the title of this journal would be more appropriate. CIK has become a global organization and participants in their conferences and papers that are submitted are global. In addition to the journal and conferences, CIK is focusing on offering post-doctoral positions at the institute. The aim is to advance young doctoral students' research capabilities through collaborations with more experienced CIK scholars. This kind of work is at the heart of the CIK mission. All parties will gain in this process and will advance knowledge. Additionally, CIK is planning to publish books in the domain of its focus. Therefore, we welcome inquires and support in these domains.

The Institute has had nine successful international conferences since its inception. These conferences have been hosted at institutions in the United States (MIT, Harvard, Hult), and internationally (Hult - UAE, American University in Cairo, and ESCA in Morocco). Several institutions of higher education have collaborated and supported these conferences. Please see the CIK website for information about these institutions. We greatly appreciate their support! *The CIK 2020 and 2021 Conferences were held **Online** and in collaboration with International Symposium on Project Management, Innovation and Sustainability (SINGEP).*

Generally, conference participants come from at least 15 different countries and 35 institutions, organizations, and companies. Please see the CIK website for details. Some of the plenary sessions had up to 150 participants. The best papers presented at these conferences have traditionally been accepted for publication in the Journal, along with additional articles by prominent scholars. The acceptance rate of *CCJ* is generally less than 20%. We aim to publish the highest quality papers after they pass through our strict review process. CIK colleagues and conference participants have proposed and suggested special issues of the journal, which are based on core topics (i.e., entrepreneurship, innovation, ethics, and sustainable development) and/or country-specific ones. Therefore, we welcome articles that meet these characteristics.

Now we welcome you to read the volume six, issue 1 (CGBP.V6.1). The journal intends to cover scholarship that pertains to the global economy, especially emerging economies. An examination of our mission may shed some light on the aim and objectives. The primary focuses of the journal are as follows:

1. To share and promote knowledge of economic, management, and development issues. Focusing on assessment, evaluation, and possible solutions helps advance these countries, which also have the largest populations. Development challenges are global; virtually all countries face challenges concerning economic development, sustainability, food and water, population, and environmental degradation. Above all, all countries are facing the Covid19 pandemic. No country gains by shunning opportunities that globalization can provide, with exception of a few countries whose leaders lack a full understanding of the opportunities that globalization can offer. To take advantage of such opportunities,

knowledge is the primary requisite. This journal aspires to contribute to this body of knowledge.

2. To encourage the generation and dissemination of knowledge by local scholars whose access to mainstream academic outlets may be limited. There are many scholars from academic, public, and private sector organizations whose first-hand knowledge of problems and solutions is not being shared for lack of an appropriate outlet for dissemination. The CGBP seeks to provide an opportunity for spreading such knowledge. The CGBP will provide a platform for established as well as younger scholars who might collaborate with them in their research. By publishing in CGBP, they could make important contributions to the body of management and development scholarship on which the journal will continue to concentrate.

Volume six, issue 1, of the CGBP contains four articles with diverse subjects. We hope you will find this volume and past issues enlightening. CGBP welcome your support and so I ask for your help in the following ways:

- *Contribute articles, case studies, and book reviews and commentaries;*
- *Encourage your colleagues to do the same;*
- *Encourage young scholars, especially those from developing and emerging economies, to submit their studies to the CGBP;*
- *Spread the word about CGBP, especially in emerging economics;*
- *Cite the articles published in this journal in your own research when applicable;*
- *Attend the annual conferences of the CIK which serve as spawning ground for articles that may ultimately be published in this journal;*
- *Give us your feedback by telling us how we can further promote and improve the journal.*
- *Review CIK aims and objectives and share with us how we could advance.*

Regards,
Editor Dr. Maling Ebrahimpour
Dean | College of Business
Alfred J. Verrecchia-Hasbro Leadership Chair
The University of Rhode Island. RI, USA

In the memory of Nancy Tagi Sagafi-nejad - former Associate Editor of CCJ

Nancy Gail Black Sagafi-nejad

[in her own words]

I was born in Wisconsin and lived in Illinois most of my early life. I attended and graduated from Northwestern University with a BA in art and then worked for a little over two years at the Isabella Stewart Gardner Museum in Boston as a curatorial assistant. After that I was fortunate enough to receive a 21-month grant to the East-West Center at the University of Hawaii where I received a master's degree in art history. Shortly thereafter I joined the U.S. Peace Corps and was sent to Pahlavi University in Shiraz, Iran with other Peace Corps Volunteers who also had their masters' degrees in various fields. It was there that I met and married Tagi, my husband of 53+ years. Upon returning to the US, I became a staff docent at the Philadelphia Museum of Art and taught art history at Rosemont College in Pennsylvania. Both our sons Jahan and David, where born at the Hospital of the University of Pennsylvania.

It was my two and one-half years in the Peace Corps that raised my social consciousness and caused me to want to contribute by working on behalf of plaintiffs in the field of employment discrimination law. After receiving my law degree from the University of Texas, Tagi, I, and our sons Jahan and David moved to Baltimore, Maryland where Tagi taught at Loyola University Maryland and I first practiced civil rights law at the Equal Employment Opportunity Commission (EEOC) in Baltimore and then in my own small solo practice of employment discrimination.

In spring 1991 I applied through my Friends meeting (Stony Run) to be the Henry J. Cadbury Scholar at Pendle Hill - the Quaker study and retreat center in Wallingford, Pennsylvania that many members of EFM are familiar with. [the picture above was taken at Pendle Hill – TS] I was fortunate to have received that scholarship and then spent the 1991 fall quarter and the 1992 winter and spring quarters in that beautiful place.

It was during that nine-month stay that I began research on the intersect of being a Quaker and a lawyer and how the two identities could occasionally, or even often, be at odds. In 2011 the research begun at Pendle Hill twenty years earlier was published by the State University of New

York (SUNY) Press as a book entitled *Friends at the Bar: A Quaker view of Law, Conflict Resolution, and Legal Reform* (SUNY Press, 2012)

We moved from south Texas where Tagi had been the Radcliffe Killam Distinguished Professor at Texas A & M International University in Laredo before his retirement in August 2015. We moved to the Pacific Northwest to be closer to our younger son David and his wife Audra and their children Cameron and Leila in Eugene, Oregon, and to our older son Jahan who lives in Oakland and works in San Francisco with his wife Kristen and their son Benjamin.
Memo: Nancy wrote the above a few weeks before she died on September 27, 2021. Can't stop the tears. ***Tagi Sagafi-nejad***

An Investigation of the Benefit of Optimal Refund over the Full Refund Strategy in Retail Market: A Numerical Study

Ali Shirzadeh Chaleshtari[a1] and Ehsan Elahi[b]

[a] *Supply Chain Management Group, College of Business, University of Rhode Island, USA*

[b] *Department of Management Science and Information Systems, College of Management, University of Massachusetts Boston, Boston, MA, USA*

ABSTRACT

Many retailers offer full refunds in the matter of product returns, which further intensifies pervasive, increasing application of this option. While the return option stimulates the market demand via rectifying customers' uncertainty regarding the value of products, it endures intense expenses to the retailer. Therefore, comparison of the full refund with the optimal refund strategy helps the retailers wisely decide whether to follow the commonly adhered strategy of full refund, or switch to the optimal refund strategy to alleviate the harms of the returns. To this end, we develop an analytical framework in this paper which can capture the impact of all major factors affecting the purchase and return behavior of customers. These factors, in addition to other commonly studied factors in the literature, include return leniency, and customers' heterogeneity. Using this framework, we characterize the probability of a customer purchasing the product and the probabilities of keeping and returning it. These probabilities in turn characterize the retailer's demand and return volume, which we use to address the optimal refund strategies of the retailers in various circumstances and specify the monetary outcomes of these strategies along with the outcomes of full refund. These results enable us the comparison between the two policies and decision making. Our analyses show under what circumstances the optimal return strategy has considerable benefit over the full refund policy. Plus, the analyses reveal the impact of various parameters on the benefit of refund strategy optimization.

Keywords: Pricing; Product Return; Return Leniency; Customer Behavior; Marketing Strategy.

[1]Corresponding author. Email addresses: shirzadeh@uri.edu (A. Shirzadeh Chaleshtari), ehsan.elahi@umb.edu (E. Elahi). We, authors of the current research paper, certify that the paper is an outcome of our independent and original work. We have duly acknowledged all the sources from which the ideas and extracts have been taken and we are responsible for any errors that may be discovered. We thank the editor of ***CYRUS Global Business Perspectives (CGBP)***, and anonymous reviewers for their careful reading of our manuscript and their many insightful comments and suggestions

INTRODUCTION

Product return acts as a demand stimulating factor in retail markets while rectifying customers' uncertainty regarding the actual products values, however at the same time, the underlying reverse supply chain of this option costs retailers a fortune. Optoro[2] estimates that returns cost retailers $400B each year in the US alone[3]. The COVID-19 pandemic has further intensified the economics of the returns while helping the shifting trend in retail market toward e-commerce, with at least three times higher return rates as compared to brick-and-mortar stores[4]. This vast economic impact and the increasing trend necessitate a careful investigation of return strategies, which is the focus of this research. More specifically, our goal is to investigate the benefit of optimal refund over the full refund strategy which is the common practice in the retail market.

Product return mainly happens due to the uncertainty in customers' valuation of a product. There are, however, many factors that affect the customer's ultimate decision, including those related to customers, such as *reservation prices*, those to products such as the acquiring cost and salvage value, and those set or regulated by the retailer such as the selling price, refund level, and return *leniency*. We construct an inclusive analytical framework, which can capture the impact of each of these major factors on product purchase and return decisions in a monopoly market with heterogeneous customer base. We characterize the probabilities of customers purchasing and returning probabilities, which are presented through a series of theorems in the following sections.

These probabilities in turn specify market demand and return frequency, using which we shape the expected profit function of the retailer. We use the framework to numerically address the optimal refund policy and compare its monetary outcomes in terms of the retailers' expected profit with the full refund policy. We conduct the numerical experiments over wide range of normalized parameters which reciprocate most of the practical circumstances. More specifically, our research contributes to the body of literature in this field by shedding lights on the following questions.

- How the price and refund level can be jointly optimized to maximize the retailers' expected policy?
- What is the benefit of return optimization over the full refund policy?
- How major factors of product return issue such as salvage value, market uncertainty, and return leniency impact the outcomes of the above questions?

Therefore, the current research contributes to the literature in three major ways; first, by presenting an analytical model which captures the impact of major factors affecting the product return in a monopoly market, while there are still some factors such as *opportunistic behavior* or *endowment effect* which the model presented in here lacks. Second, via analytically modeling the factor of

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[2] www.optoro.com
[3] Creating Value from Returns this Holiday Season https://rla.org/media/article/view?id=1179
[4] E-commerce Product Return Rate – Statistics and Trends [Infographic] https://www.invespcro.com/blog/ecommerce-product-return-rate-statistics/

return leniency via incorporating return efficiency factor in the model, and third, by enabling comparison between optimal refund policy and full refund policy of retailers which is novel in the literature while having extensive practical interest, as the full refund policy is commonly adhered strategies in the retail markets although it may result in lower profit levels for the retailers. Competition among the retailers in the market structures other than monopoly is the major simplification of the current research.

Our analyses show that using an optimal refund policy is particularly beneficial over the full refund policy when the cost of acquiring the product and market uncertainty are high and salvage value is low. In addition, the study shows that relatively lower optimal refund levels are associated with the higher benefits of optimal refund over the full refund policy. Finally, the results shows that the majors factors assumed in this study are drivers of the results.

The rest of this paper is organized as the following. Section 2 reviews the related literature. In section 3, we formulate the problem and present the analytical relations that governs the purchase and return decisions. In section 4, we present the results of our numerical experiments, and in section 5, we conclude with a few remarks.

LITERATURE REVIEW

To characterize the benefit of using the optimal return strategy, our research models the behavior of heterogeneous customers in a monopoly market while capturing the impact of major related factors such as customers' return efficiency, product salvage value, and hassle cost of return. Although the topic of product return has been studied in the literature, to the best of our knowledge, no research has studied all these factors together and addressed the relative value of optimal refund strategy compared to the more common practice of full refund. Here we briefly review the related literature.

There are researchers who study joint optimization of pricing and return in the context of supply chain management, such as Pasternack (1985), Lau and Lau (1999), Pasternack (2008), Aviv and Pazgal (2008), Ding and Chen (2008), Su (2009), Xiao et al. (2010), Samatli-Pac et al. (2018), Li et al. (2019), De Giovanni and Zaccour (2019), Tian and Zhang (2019), and Li et al. (2020). In comparison, we focus on the joint optimization of price and refund level in the context of revenue management, excluding inventory management.

Customers' uncertainty about product value can be considered as the main reason behind most product returns. This factor is the source of customers heterogeneity in the market. Most of analytical studies on product return management strategies lack customers heterogeneity in the market, such as Matthews and Persico (2007) and Akturk et al. (2020). Shulman et al. (2009) consider a simplified heterogeneity condition when only two types of customers exist in the market; customers who may find the product useful and customers who may not. Becher et al.

(2018) consider another form of heterogeneity. In their model, they consider the possibility of a fixed amount of reduction in the customers' valuation of the product after the purchase. In comparison, the randomness in customers' valuation of products in our model let the actual valuation attain any value above or below the expected value within the defined range.

Opportunistic behavior is another source of product return in retail markets when customer purchase the product with the intention of return it after using it for some while. This behavior has been studied in some research works including Hess et al. (1996), Su and Zhang (2008), Shang et al. (2017), and Altug et al. (2021). In comparison, our research ignores opportunistic returns and studies uncertainty in product valuation as the main source of returns.

Most studies define return leniency based on the seller's return policy terms. See for example Bower and Maxham III (2012), Kim and Wansink (2012), Pei et al. (2014), Khouja et al. (2019), and Cui et al. (2020). Suwelack and Krafft (2012) note there are significant differences in retailers' return policy terms. Davis et al. (1998), Heiman et al. (2001), and Janakiraman et al. (2016) each classify the return policies in terms of different restriction factors, such as time, money, and scope. Matthews and Persico (2007) and Shulman et al. (2009) define return leniency in terms of the return hassle cost. In our research, however, our model captures the impact of return leniency through a combination of hassle cost and return efficiency, which we define as the percentage of customers who intend to return the product and manage to do so. Lower return efficiency could be, in part, the result of higher hassle cost as well as all other factors that make the return more challenging (low return leniency).

Customers' hesitancy to return a product supports the endowment effect theory which considers a gap between valuation of a product before and after a purchase (Knetsch (1989); Kahneman et al. (1990); Thaler (1980)). Some studies of product return management strategies include this factor, such as Becher et al. (2020). They study the impact of endowment effect on the jointly optimized retailer's price and return fee decisions in a homogeneous market. In comparison, our research focuses on the optimal strategies of the retailer in a heterogeneous market.

ANALYTICAL FRAMEWORK

A monopolist retailer sells a product with unit price p. Each potential customer i has a unit demand for the product and has his/her own reservation price for the product, R_i, which reflects how much the customer values the product. R_i is a uniformly distributed random variable over $\left[\overline{R}_i - \delta, \overline{R}_i + \delta\right]$, where $\overline{R}_i$ is the mean of this distribution. From the retailer's perspective, $\overline{R}_i$ itself is also a uniformly distributed random variable over the interval $[L, U]$, which reflects the variation in mean reservation prices of retailer's pool of customers. Before buying the product, customer i is only aware of the distribution of his/her own reservation price for the product, not the exact amount of R_i. After buying the product, the actual amount of R_i is revealed to the customer. This can happen often in online shopping when the customer cannot examine the product up-close before the purchase. This can also happen in the purchase of highly featured products, or when the product should match other items, such as decorative products or pieces of clothing, or simply when the customer wants to buy a new type of product or brand. Modeling customers' uncertainty in this fashion is in accordance with the literature, see for example, Matthews and Persico (2007) and Shulman et al. (2009).

Knowing the actual amount of R_i, the utility of buying and keeping the product is revealed for the customer, which would be R_i-p. Obviously, the utility can be negative when the revealed reservation price is less than the product price. Having bought the product, customer i has the choice to return the product if she is not satisfied with it. Upon return, the customer endures some form of hassle cost h, and receives the refund value of βp, where $0 \le \beta \le 1$. The cases of $\beta = 1$ and $\beta = 0$ refer to full refund and no refund policies, respectively. To decide whether to return the product or not, the customer checks if she would get more utility by returning the product or by keeping it. Even when return is the favorable option, the customer might fail to do so due to different reasons, such as losing the receipt, missing the deadline, or simply forgetting. To consider these possibilities, we add to our model a return efficiency factor □ $(0 \le \alpha \le 1)$, which reflects the probability of returning the product when return is preferable. Higher return leniency corresponds with less hassle cost and possibly higher return efficiency factor. So the overall utility of purchasing, and possibly returning, the product is

$$U_i = \begin{cases} R_i - p & \begin{cases} if\ v_r \le R_i \text{ .... } customer\ keeps\ the\ product \\ if\ v_r > R_i \text{ .... } customer\ keeps\ the\ product\ with\ a\ probability\ of\ (1-\alpha) \end{cases} \\ (\beta - 1)p - h \quad if\ v_r > R_i \text{ .... } customer\ returns\ the\ product\ with\ a\ probability\ of\ \alpha \end{cases} \tag{1}$$

Proposition 1. *Customer i purchases the product if and only if* $\overline{R}_i > R_{\min}$*, where:*

$$R_{\min} = \begin{cases} r_2 & if\ p \le v_r + \delta \\ p & if\ p > v_r + \delta \end{cases} \quad \text{and} \quad r_2 = \frac{-2\delta + \alpha(\delta + v_r) + 2\sqrt{\delta\alpha(p - v_r) + \delta^2(1-\alpha)}}{\alpha}$$

Proof of all propositions are available in the appendix.

Based on these results, the purchase probability of any randomly selected customer is:

$$\Pr^{Purchase} = \frac{U - R_{\min}}{U - L} \tag{2}$$

Proposition 2. *The probability of purchasing and keeping the product,* $\Pr^{Keep}$*, and the probability of purchasing and returning the product,* $\Pr^{Keep}$ *are as stated below:*

$$\Pr^{Keep} = \frac{U - p}{U - L} \tag{3}$$

$$\Pr^{Return} = \frac{p - R_{\min}}{U - L} \quad (4)$$

We now turn our attention to the problem from the retailer's point of view. For the retailer, there exist a total expense of *c* to acquire and sell each unit of the product, and it sells each unit of the returned product with a salvage value of *s*. To avoid selling and return arbitrage, we assume that $s < c$, which is a common assumption in the literature (see e.g., Shulman et al. (2009), Matthews and Persico (2007)). So, the retailer's profit from a given customer *i*, π_i can be stated as:

$$\pi_i = \begin{cases} p - c \quad \textit{when customer buys and keeps the product} \\ (1-\beta)\, p - c + s \quad \textit{when customer buys and returns the product} \\ 0 \quad \textit{otherwise} \end{cases} \qquad (5)$$

We can then state the retailer's expected profit per customer as follows.

$$E_\pi = (p-c)P^{Keep} + \left((1-\beta)\,p + s - c\right)P^{\mathrm{Return}} \quad (6)$$

The joint optimal values of selling price, p^*, and refund level, □*, that maximize the retailers' expected profit, as stated in equation 6, are not mathematically tractable[5]. Nevertheless, to shed light on the behavior of retailer's optimal expected profit (based on p^* and □*) for different values of problem parameters, section 3 presents a set of numerical experiments for a wide range of problem parameters.

NUMERICAL EXPERIMENTS

In this section, via numerical experiments, we investigate how much extra profit the retailer gains by implementing the optimal refund level instead of using a more commonly adhered full refund strategy. We use normalized parameters' values consistent with the literature (see, e.g., Shulman et al. (2009), Matthews and Persico (2007)) as presented in table 1.

Table 1. Values of basic parameters for the numerical experiments

Parameter	Symbol	Low value	High value
Reservation price uncertainty	□	$0.1\times(U-L)$	$0.5\times(U-L)$
Cost of acquiring the product	c	$\frac{1}{3}\times\frac{U+L}{2}$	$\frac{U+L}{2}$
Return Leniency (Hassle cost of return,Return efficiency)	(h,α)	$(0.1\times c, 0.5)$	$(0.01\times c, 0.9)$
Product salvage value	s	$0.1\times c$	$0.9\times c$

[5] This optimization problem is in nature *multivariable non-separable nonlinear programming (NLP)*. No polynomial-time exact algorithm can be devised for this problem as it is categorized under *NP-hard* complexity level Hochbaum, D. S. (2007). Complexity and algorithms for nonlinear optimization problems. *Annals of Operations Research*, *153*(1), 257-296.

Figure 1 demonstrates, a set of graphs in which the (left-hand-side) vertical axes are the relative decrease in retailer's expected profit (in percentage) when the retailer uses the commonly used full refund strategy, □□□□□□instead of the optimal refund level, □□□. We call this relative decrease *refund optimization efficiency*. In this figure, the optimal refund level, □□□, has been illustrated on the second vertical axes (right-hand-side).

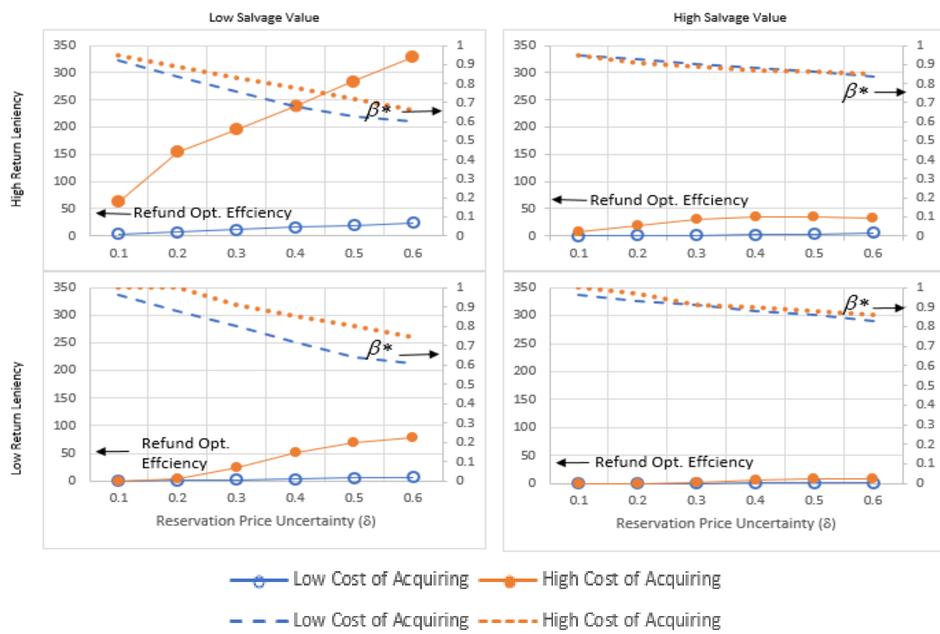

Figure 1. Return optimization efficiency (%) and optimal refund level versus customers' uncertainty

We can observe the following trends in the results of these numerical experiments.

- The highest benefit of refund optimization happens when return leniency is high (easy return) and the product salvage value is low. That is, when a large proportion of customers return the product, while the returned product is not very valuable to the retailer (low salvage value) the retailer gains more from using the optimal refund level rather than full refund strategy. This is particularly more pronounced for more expensive products (high acquiring cost). These are all conditions that make the product return more costly for the retailer.
- In the majority of cases, an increase in customers' uncertainty about his/her reservation price makes the refund optimization more valuable. When the salvage value is high, the benefit of using the optimal refund level becomes less sensitive to the customers' uncertainty.
- At the extreme cases of high salvage value, inexpensive products, and low return leniency, the optimal refund strategy and full refund strategy almost always result in the same expected profit for the retailer, regardless of customers' reservation price uncertainty. This

is because, under these conditions, customers rarely return the product while the returns are not very costly due to high salvage value.

- As one can expect, a larger refund optimization efficiency corresponds to a smaller value of β*. That is, when the optimal refund level gets farther from $\beta = 1$, the retailer gains more by using this optimal value rather than using the full refund strategy.

CONCLUSIONS

In this paper we presented an analytical framework to model customers' purchase and return decisions in a monopoly market. The framework captures for the first time, to the best of our knowledge, the impact of several influential factors altogether, among which customer heterogeneity, and return leniency are novel in the product return literature (revenue management context). We specified the conditions under which a given customer makes a purchase. This helped us determine the probabilities of purchasing, keeping, and returning the product which in turn enabled us to calculate the optimal price and refund level through numerical calculations.

We conducted inclusive numerical experiments using normalized problem parameters to help us with generalization of the results. Through these numerical experiments, we studied the benefit of the refund level optimization over the commonly adhered strategy of full refund. Our analysis showed, among other results, lower uncertainty lead to lower benefit of refund optimization. This is particularly more pronounced when the product is more costly for the retailer to acquire.

The research of this paper can be further extended in many different directions. To mention a few, different market structures other than monopoly can be studied, such as duopoly or oligopoly, the pricing and return strategies can be examined for the product bundles, and the impact of product quality, price adjustment strategy, and return insurance policies can be jointly studied with the current subject matter. In addition, the impact of retailers' refund policies on the probabilities of purchasing and returning the product, or equivalently, market demand and return frequency, can be investigated, particularly when market structure is other than monopoly.

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APPENDIX - PROOF OF PROPOSITIONS

Proposition 1

Conditioning on the value of $\overline{R}_i$, we can determine the expected utility of buying the product for customer i as the following.

When $\overline{R}_i < v_r - \delta$, the expected utility is

$$E(U_i) = E\left(R_i - p \middle| R_i > v_r\right)P\left(R_i > v_r\right) + E\left(R_i - p \middle| R_i \le v_r\right)P\left(R_i \le v_r\right)(1-\alpha) +$$
$$E\left((\beta-1)p - h \middle| R_i \le v_r\right)P\left(R_i \le v_r\right)\alpha = E\left(R_i - p \middle| R_i > v_r\right)0 +$$
$$E\left(R_i - p \middle| R_i \le v_r\right).1.(1-\alpha) + E\left((\beta-1)p - h \middle| R_i \le v_r\right).1.\alpha = \left(\overline{R}_i - p\right)(1-\alpha) + \left((\beta-1)p - h\right)\alpha,$$

which is always non-positive. So, in this case, customer does not purchase the product.

When $v_r - \delta \le \overline{R}_i \le v_r + \delta$, the expected utility is

$$E(U_i) = E\left(R_i - p \middle| R_i > v_r\right)P\left(R_i > v_r\right) + E\left(R_i - p \middle| R_i \le v_r\right)P\left(R_i \le v_r\right)(1-\alpha) +$$
$$E\left((\beta-1)p - h \middle| R_i \le v_r\right)P\left(R_i \le v_r\right)\alpha$$
$$= \left(\frac{\overline{R}_i + \delta + \beta p - h}{2} - p\right)\left(\frac{\overline{R}_i + \delta - \beta p + h}{2\delta}\right) + \left(\frac{\overline{R}_i - \delta + \beta p - h}{2} - p\right)\left(\frac{\beta p - h - \overline{R}_i + \delta}{2\delta}\right)(1-\alpha) +$$
$$\left((\beta-1)p - h\right)\left(\frac{\beta p - h - \overline{R}_i + \delta}{2\delta}\right)\alpha$$
$$= \frac{1}{4\delta}\begin{pmatrix} \left(\overline{R}_i + \delta + \beta p - h - 2p\right)\left(\overline{R}_i + \delta - \beta p + h\right) + \left(\overline{R}_i - \delta + \beta p - h - 2p\right)\left(\beta p - h - \overline{R}_i + \delta\right)(1-\alpha) + \\ \left(2(\beta-1)p - 2h\right)\left(\beta p - h - \overline{R}_i + \delta\right)\alpha \end{pmatrix}$$

In this situation, the expected utility is a convex, quadratic function of $\overline{R}_i$. We need to see whether this function is positive for any value of $\overline{R}_i$ over $\left[v_r - \delta, v_r + \delta\right]$. Determinant of this quadratic function is

$$\Delta = 16p\delta\alpha(1-\beta) + 16\alpha\delta h + 16\delta^2(1-\alpha),$$

which is always nonnegative since $0 \le \alpha \le 1$, $0 \le \beta \le 1$, $p > 0$, $\delta > 0$, $h \ge 0$. The determinant may be zero if we had either $\delta = 0$ or, $p = \dfrac{-h - \delta(1-\alpha)}{1-\beta}$ which is negative value. Since neither of these can happen, the determinant is always positive. So, the quadratic function has two roots, say r_1 and r_2 (supposing $r_1 < r_2$), where

$$r_1 = \frac{-2\delta + \alpha(\delta + \beta p - h) - 2\sqrt{\delta\alpha(p - \beta p + h) + \delta^2(1-\alpha)}}{\alpha}$$

$$r_2 = \frac{-2\delta + \alpha(\delta + \beta p - h) + 2\sqrt{\delta\alpha(p - \beta p + h) + \delta^2(1-\alpha)}}{\alpha}$$

The quadratic function is positive for $\overline{R}_i < r_1$ and $r_2 < \overline{R}_i$. Now we see which parts of these ranges may lay in $[v_r - \delta, v_r + \delta]$. The minimum point of the quadratic function is

$$\overline{R}_i^* = \frac{-4\delta + (2\delta + 2\beta p - 2h)\alpha}{2\alpha} = \delta + \beta p - h - \frac{2\delta}{\alpha} = \delta + v_r - \frac{2\delta}{\alpha},$$

which equals $v_r - \delta$ when $\alpha = 1$, and is smaller than $v_r - \delta$ when $\alpha < 1$. So in both cases, the lower root r_1 is always less than $v_r - \delta$, and the larger root r_2 may lay in $[v_r - \delta, v_r + \delta]$ or not. If we assume $r_2 < v_r - \delta$, we will have,

$$\frac{-2\delta + \alpha(\delta + \beta p - h) + 2\sqrt{\delta\alpha(p - \beta p + h) + \delta^2(1-\alpha)}}{\alpha} < \beta p - h - \delta$$

$$\to \alpha(p - \beta p + h) < \delta(-\alpha + \alpha^2) \to (p - \beta p + h) < \delta(-1 + \alpha) \to (p - \beta p + h) < 0$$

$$\to p < \beta p - h,$$

which cannot be true. Therefore $r_2 \ge v_r + \delta$. Now we see when $r_2 < v_r + \delta$. If we assume this, we will have,

$$\frac{-2\delta + \alpha(\delta + \beta p - h) + 2\sqrt{\delta\alpha(p - \beta p + h) + \delta^2(1-\alpha)}}{\alpha} \le \beta p - h + \delta$$

$$\to -2\delta + \alpha(\delta + \beta p - h) + 2\sqrt{\delta\alpha(p - \beta p + h) + \delta^2(1-\alpha)} \le (\beta p - h + \delta)\alpha$$

$$\to p - \beta p + h \le \delta$$

$$\to p \le v_r + \delta$$

So $p \le v_r + \delta$ corresponds to the situation that $r_2 < v_r + \delta$, and subsequently there is positive purchase probability. Therefore, in case that $v_r - \delta \le \overline{R}_i \le v_r + \delta$, if we have $p \le v_r + \delta$, then those customers purchase the product whose mean reservation prices are larger than r_2.

When $v_r + \delta \le \overline{R}_i$, the expected utility would be

$$E(U_i) = E\left(R_i - p \middle| R_i > v_r\right) P\left(R_i > v_r\right) + E\left(R_i - p \middle| R_i \le v_r\right) P\left(R_i \le v_r\right)(1-\alpha) +$$
$$E\left((\beta-1)p - h \middle| R_i \le v_r\right) P\left(R_i \le v_r\right)\alpha = \left(\overline{r}_i - p\right).1 + E\left(R_i - p \middle| R_i \le v_r\right).0.(1-\alpha) +$$
$$E\left((\beta-1)p - h \middle| R_i \le v_r\right).0.\alpha = \left(\overline{R}_i - p\right),$$

which is positive if $p < \overline{R}_i$.

Therefore, in overall, if $p \le v_r + \delta$, customer purchases the product if his mean reservation price is not lower than the price, and if $v_r + \delta < p$, the customer purchases the product if his mean reservation price is higher than the price.

$$= \frac{1}{4\delta}\begin{pmatrix} \alpha \overline{R}_i^{\,2} + \\ \left(4\delta + (-2\beta p - 2\delta + 2h)\alpha\right)\overline{R}_i \\ +(\delta + \beta p - h - 2p)(\delta - \beta p + h) + (-\delta + \beta p - h - 2p)(\beta p - h + \delta)(1-\alpha) + \\ \left(2(\beta-1)p - 2h\right)(\beta p - h + \delta)\alpha \end{pmatrix}$$

Proposition 2

$$\Pr^{Keep} = P\left(keeping \middle| purchase\right) P\left(purchase\right) =$$
$$\int_{\forall \overline{r}_i} P\left(keeping \middle| purchase, \overline{R}_i = \overline{r}_i\right) P\left(purchase \middle| \overline{R}_i = \overline{r}_i\right) f_{\overline{R}_i}\left(\overline{r}_i\right) d\overline{r}_i$$
$$= \int_L^{\beta p - h - \delta} P\left(keeping \middle| purchase, \overline{R}_i = \overline{r}_i\right) P\left(purchase \middle| \overline{R}_i = \overline{r}_i\right) f_{\overline{R}_i}\left(\overline{r}_i\right) d\overline{r}_i$$
$$+ \int_{\beta p - h - \delta}^{\beta p - h + \delta} P\left(keeping \middle| purchase, \overline{R}_i = \overline{r}_i\right) P\left(purchase \middle| \overline{R}_i = \overline{r}_i\right) f_{\overline{R}_i}\left(\overline{r}_i\right) d\overline{r}_i$$
$$+ \int_{\beta p - h + \delta}^{U} P\left(keeping \middle| purchase, \overline{R}_i = \overline{r}_i\right) P\left(purchase \middle| \overline{R}_i = \overline{r}_i\right) f_{\overline{R}_i}\left(\overline{r}_i\right) d\overline{r}_i$$
$$= \int_L^{\beta p - h - \delta} P\left(keeping \middle| purchase, \overline{R}_i = \overline{r}_i\right).0.f_{\overline{R}_i}\left(\overline{r}_i\right) d\overline{r}_i$$
$$+ \int_{\beta p - h - \delta}^{\min(r_2, \beta p - h + \delta)} P\left(keeping \middle| purchase, \overline{R}_i = \overline{r}_i\right) 0\, f_{\overline{R}_i}\left(\overline{r}_i\right) d\overline{r}_i$$
$$+ \int_{\min(r_2, \beta p - h + \delta)}^{\beta p - h + \delta} P\left(keeping \middle| purchase, \overline{R}_i = \overline{r}_i\right).1.f_{\overline{R}_i}\left(\overline{r}_i\right) d\overline{r}_i$$
$$+ \int_{\beta p - h + \delta}^{\max(\beta p - h + \delta, p)} P\left(keeping \middle| purchase, \overline{R}_i = \overline{r}_i\right).0.f_{\overline{R}_i}\left(\overline{r}_i\right) d\overline{r}_i$$
$$+ \int_{\max(\beta p - h + \delta, p)}^{U} P\left(keeping \middle| purchase, \overline{R}_i = \overline{r}_i\right).1.f_{\overline{R}_i}\left(\overline{r}_i\right) d\overline{r}_i$$
$$= \int_{\min(r_2, \beta p - h + \delta)}^{\beta p - h + \delta} \left(\frac{\overline{r}_i + \delta - \beta p + h}{2\delta} + (1-\alpha)\frac{\beta p - h - \overline{r}_i + \delta}{2\delta}\right).\frac{1}{U-L} d\overline{r}_i + \int_{\max(\beta p - h + \delta, p)}^{U} \frac{1}{U-L} d\overline{r}_i$$

$$=\int_{\min(r_2,\beta p-h+\delta)}^{\beta p-h+\delta}\left(1-\alpha\frac{\beta p-h-\bar{r}_i+\delta}{2\delta}\right).\frac{1}{U-L}d\bar{r}_i+\int_{\max(\beta p-h+\delta,p)}^{U}\frac{1}{U-L}d\bar{r}_i$$

$$=\left(\left(1-\alpha\frac{\beta p-h+\delta}{2\delta}\right)\bar{r}_i+\alpha\frac{\bar{r}_i^{\,2}}{4\delta}\right).\frac{1}{U-L}\Bigg|_{r_2}^{\beta p-h+\delta}+\frac{1}{U-L}\left(U-\left(\beta p-h+\delta\right)\right)$$

$$=\left(\left(1-\alpha\frac{\beta p-h+\delta}{2\delta}\right)\left(\beta p-h+\delta-r_2\right)+\alpha\frac{\left(\beta p-h+\delta\right)^2-\left(r_2\right)^2}{4\delta}\right).\frac{1}{U-L}+\frac{1}{U-L}\left(U-\left(\beta p-h+\delta\right)\right)$$

$$=\left(\left(1-\alpha\frac{\beta p-h+\delta}{2\delta}\right)\left(\beta p-h+\delta-r_2\right)+\frac{\alpha}{4\delta}\left(\beta p-h+\delta-r_2\right)\left(\beta p-h+\delta+r_2\right)\right).\frac{1}{U-L}$$

$$+\frac{1}{U-L}\left(U-\left(\beta p-h+\delta\right)\right)$$

$$=\left(\left(1-\alpha\frac{\beta p-h+\delta}{2\delta}+\frac{\alpha}{4\delta}\left(\beta p-h+\delta+r_2\right)\right)\left(\beta p-h+\delta-r_2\right)\right).\frac{1}{U-L}+\frac{1}{U-L}\left(U-\left(\beta p-h+\delta\right)\right)$$

$$=\frac{1}{U-L}\left(\left(\left(1-\alpha\frac{\beta p-h+\delta}{4\delta}+\frac{\alpha}{4\delta}r_2\right)\left(\beta p-h+\delta-r_2\right)\right)+U-\left(\beta p-h+\delta\right)\right)$$

$$=\frac{1}{U-L}\left(\begin{array}{l}\left(\begin{array}{l}\left(1-\alpha\frac{\beta p-h+\delta}{4\delta}+\frac{\alpha}{4\delta}\frac{-2\delta+\alpha\left(\delta+\beta p-h\right)+2\sqrt{\delta\alpha\left(p-\beta p+h\right)+\delta^2\left(1-\alpha\right)}}{\alpha}\right)\\ \left(\beta p-h+\delta-\frac{-2\delta+\alpha\left(\delta+\beta p-h\right)+2\sqrt{\delta\alpha\left(p-\beta p+h\right)+\delta^2\left(1-\alpha\right)}}{\alpha}\right)\end{array}\right)\\ +U-\left(\beta p-h+\delta\right)\end{array}\right)$$

$$=\frac{1}{U-L}\left(\frac{\delta^2-\left(\delta\alpha\left(p-\beta p+h\right)+\delta^2\left(1-\alpha\right)\right)}{\delta\alpha}+U-\left(\beta p-h+\delta\right)\right)$$

$$=\frac{1}{U-L}\left(-\left(p-\beta p+h\right)+\delta+U-\left(\beta p-h+\delta\right)\right)=\frac{U-p}{U-L}$$

$$\Pr^{Return}=P\left(returning\,|\,purchase\right)P\left(purchase\right)=$$

$$\int_{\forall\bar{r}_i}P\left(returning\,|\,purchase,\bar{R}_i=\bar{r}_i\right)P\left(purchase\,\middle|\,\bar{R}_i=\bar{r}_i\right)f_{\bar{R}_i}\left(\bar{r}_i\right)d\bar{r}_i$$

$$=\int_{L}^{\beta p-h-\delta}P\left(returning\,|\,purchase,\bar{R}_i=\bar{r}_i\right)P\left(purchase\,\middle|\,\bar{R}_i=\bar{r}_i\right)f_{\bar{R}_i}\left(\bar{r}_i\right)d\bar{r}_i$$

$$+\int_{\beta p-h-\delta}^{\beta p-h+\delta}P\left(returning\,|\,purchase,\bar{R}_i=\bar{r}_i\right)P\left(purchase\,\middle|\,\bar{R}_i=\bar{r}_i\right)f_{\bar{R}_i}\left(\bar{r}_i\right)d\bar{r}_i$$

$$+\int_{\beta p-h+\delta}^{U}P\left(returning\,|\,purchase,\bar{R}_i=\bar{r}_i\right)P\left(purchase\,\middle|\,\bar{R}_i=\bar{r}_i\right)f_{\bar{R}_i}\left(\bar{r}_i\right)d\bar{r}_i$$

$$= \int_{L}^{\beta p - h - \delta} P\left(returning \,\middle|\, purchase, \bar{R}_i = \bar{r}_i\right).0.f_{\bar{R}_i}\left(\bar{r}_i\right) d\bar{r}_i$$

$$+ \int_{\beta p - h - \delta}^{\min(r_2, \beta p - h + \delta)} P\left(returning \,\middle|\, purchase, \bar{R}_i = \bar{r}_i\right) 0\, f_{\bar{R}_i}\left(\bar{r}_i\right) d\bar{r}_i$$

$$+ \int_{\min(r_2, \beta p - h + \delta)}^{\beta p - h + \delta} P\left(returning \,\middle|\, purchase, \bar{R}_i = \bar{r}_i\right).1.f_{\bar{R}_i}\left(\bar{r}_i\right) d\bar{r}_i$$

$$+ \int_{\beta p - h + \delta}^{\max(\beta p - h + \delta, p)} P\left(returning \,\middle|\, purchase, \bar{R}_i = \bar{r}_i\right).0.f_{\bar{R}_i}\left(\bar{r}_i\right) d\bar{r}_i$$

$$+ \int_{\max(\beta p - h + \delta, p)}^{U} P\left(returning \,\middle|\, purchase, \bar{R}_i = \bar{r}_i\right).1.f_{\bar{R}_i}\left(\bar{r}_i\right) d\bar{r}_i$$

$$= \int_{\min(r_2, \beta p - h + \delta)}^{\beta p - h + \delta} P\left(returning \,\middle|\, purchase, \bar{R}_i = \bar{r}_i\right).1.f_{\bar{R}_i}\left(\bar{r}_i\right) d\bar{r}_i$$

$$+ \int_{\max(\beta p - h + \delta, p)}^{U} P\left(returning \,\middle|\, purchase, \bar{R}_i = \bar{r}_i\right).1.f_{\bar{R}_i}\left(\bar{r}_i\right) d\bar{r}_i$$

$$= \int_{\min(r_2, \beta p - h + \delta)}^{\beta p - h + \delta} \left(\alpha \frac{\beta p - h - \bar{r}_i + \delta}{2\delta}\right).\frac{1}{U - L} d\bar{r}_i + \int_{\max(\beta p - h + \delta, p)}^{U} 0.\frac{1}{U - L} d\bar{r}_i$$

$$= \frac{1}{2\delta(U - L)} \alpha \left(\frac{-\bar{r}_i^2}{2} + (\delta + \beta p - h)\bar{r}_i\right)\Bigg|_{\min(r_2, \beta p - h + \delta)}^{\beta p - h + \delta}$$

When $\min(r_2, \beta p - h + \delta) = \beta p - h + \delta$, the above integral and subsequently, $\Pr^{Return}$ are zero, otherwise, it equals

$$\Pr^{Return} = \frac{1}{2\delta(U - L)} \alpha \left(\frac{-\bar{r}_i^2}{2} + (\delta + \beta p - h)\bar{r}_i\right)\Bigg|_{r_2}^{\beta p - h + \delta}$$

$$= \frac{1}{2\delta(U - L)} \alpha \left(\frac{-(\beta p - h + \delta)^2}{2} + (\delta + \beta p - h)^2\right) - \frac{1}{2\delta(U - L)} \alpha \left(\frac{-r_2^2}{2} + (\delta + \beta p - h) r_2\right)$$

$$= \frac{1}{2\delta(U - L)} \alpha \left(-\frac{(\beta p - h + \delta - r_2)(\beta p - h + \delta + r_2)}{2} + (\delta + \beta p - h)(\beta p - h + \delta - r_2)\right)$$

$$= \frac{1}{2\delta(U - L)} \alpha \left(\left(\delta + \beta p - h - \frac{(\beta p - h + \delta + r_2)}{2}\right)(\beta p - h + \delta - r_2)\right)$$

$$= \frac{1}{2\delta(U - L)} \alpha \left(\left(\frac{\beta p - h + \delta - r_2}{2}\right)(\beta p - h + \delta - r_2)\right) = \frac{1}{4\delta(U - L)} \alpha (\beta p - h + \delta - r_2)^2$$

$$= \frac{1}{\delta(U - L)} \alpha \left(\frac{\delta - \sqrt{\delta\alpha(p - \beta p + h) + \delta^2(1 - \alpha)}}{\alpha}\right)^2$$

$$= \frac{1}{\delta(U - L)} \alpha \left(\frac{\delta^2 + \delta\alpha(p - \beta p + h) + \delta^2(1 - \alpha) - 2\delta\sqrt{\delta\alpha(p - \beta p + h) + \delta^2(1 - \alpha)}}{\alpha^2}\right)$$

$$
=\frac{1}{(U-L)}\left(\frac{\delta+\alpha\left(p-\beta p+h\right)+\delta\left(1-\alpha\right)-2\sqrt{\delta\alpha\left(p-\beta p+h\right)+\delta^{2}\left(1-\alpha\right)}}{\alpha}\right)
$$

$$
=\frac{1}{(U-L)}\left(p-\frac{-2\delta+\alpha\left(\beta p-h\right)+\delta\left(\alpha\right)+2\sqrt{\delta\alpha\left(p-\beta p+h\right)+\delta^{2}\left(1-\alpha\right)}}{\alpha}\right)=\frac{1}{(U-L)}\left(p-r_{2}\right)
$$